UNIT-3

TRANSPORTATION PROBLEM

Mathematical Formulation

Q:TRANSPORTATION PROBLEM IS A SPECIAL CASE OF LPP.

Consider a transportation problem with m origins (rows) and n destinations (columns). Let C_{ij} be the cost of transporting one unit of the product from the i^{th} origin to j^{th} destination. a_i the quantity of commodity available at origin I, b_j the quantity of commodity needed at destination j. x_{ij} is the quantity transported from i^{th} origin to j^{th} destination. This can be in the tabular form as follows

Destinations

Sources

	1	2	3	…….	n	Capacity
1	X_{11} C_{11}	X_{12} C_{12}	X_{13} C_{13}	……	X_{1n} C_{1n}	a_1
2	X_{21} C_{21}	X_{22} C_{22}	X_{23} C_{23}	……	X_{2n} C_{2n}	a_2
3	X_{31} C_{31}	X_{32} C_{32}	X_{33} C_{33}	…..	X_{3n} C_{3n}	a_3
m	X_{m1} C_{n1}	X_{m2} Cn_1	X_{m3} C_{m3}	…..	X_{mn} C_{mn}	a_m
Demand	b_1	b_2	b_3	…..	b_n	$\sum a_i = \sum b_j$

The linear programming model representing the transportation problem is given by

$$\text{Minimize } Z = \sum\sum c_{ij}x_{ij}$$

Subject to the constraints

$\sum x_{ij}=a_i$ $\quad i=1,2,\ldots\ldots n$ $\quad$ (row sum)

$\sum x_{ij} =b_j$ $\quad j= 1,2,\ldots\ldots n$ $\quad$ (column sum)

$X_{ij}\geq 0$ $\quad$ for all i and j

The given transportation problem is said to be balanced if

$$\sum a_i = \sum b_j$$

i. e. If the total supply is equal to the total demand.

DEFINITIONS

Feasible Solution Any set of non-negative allocations ($x_{ij} >0$) which satisfies the row and column sum is called a feasible solution.

Basic Feasible Solution A feasible solution is called a basic feasible solution if the number of non-negative allocations is equal to m+n-1 where m is the number of rows, n the number of columns in a transportation table.

OPTIMAL (OPTIMUM) SOLUTION

Optimal solution is a feasible solution which minimizes the total cost.

The solution of a transportation problem can be obtained in two stages namely initial solution and optimum solution.

Initial solution can be obtained by using any one of the three methods viz.,

(i) **Northwest corner rule (NWCR)**
(ii) **Least cost method or Matrix minima method**
(iii) **Vogel's approximation method (VAM)**

i) NOTH WEST CORNER RULE

Step 1 Starting with the cell at the upper left corner of the transportation matrix we allocate as much as possible so that either the capacity of the first row is exhausted or the destination requirement of the first column is satisfied ie., $x_{11}= \min (a_1,b_1)$.

Step 2 If $b_1>a_1$, we move down vertically to the second row and make the second allocation of magnitude $x_{21}=\min (a_2 , b_1-x_{11})$ in the cell (2,1)

If $b_1<a_1$, move right horizontally to the second column and make the second allocation of magnitude $x_{12}= \min (a_1-x_{11} ,b_2)$ in the cell (1,2)

If $b_1 =a_1$ there is a tie for the second allocation. We make the second allocation of magnitude.

$X_{12}=\min (a_1-a_1, b_2) =0$ in cell (1,2)

Or $X_{21} = \min (a_2, b_1-b_1) = 0$ in the cell (2,1)

Step 3 Repeat steps 1 and 2 moving down towards the lower right corner of the transportation table until all the rim requirements are satisfied.

ii) Least Cost or Matrix Minima Method

Step 1 Determine the smallest cost in the cost matrix of the transportation table. Let it be C_{ij}. Allocate $x_{ij} = \min (a_i,b_j)$ in the cell (i,j)

Step 2 If $x_{ij}=a_i$ cross off the i^{th} row of the transportation table and decrease b_j by a_i, then go to step 3.

If $x_{ij}= b_j$ cross off the j^{th} column of the transportation table and decrease a_i by b_j. Go to step 3.

If $x_{ij}=a_i=b_j$ cross off either the i^{th} row or the j^{th} column but not both.

Step 3 Repeat steps 1 and 2 for the resulting reduced transportation table until all the rim requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minimum.

iii) Vogel's Approximation Method (VAM)

The steps involved in this method for finding the initial solution are as follows.

Step 1 Find the penalty cost, namely the difference between the smallest and next smallest costs in each row and column.

Step 2 Among the penalties a s found in step (1) choose the maximum penalty. If this maximum penalty is more than one choose any one arbitrarily.

Step 3 In the selected row or column as by step (2) find out the cell having the least cost. Allocate to this cell as much as possible depending on the capacity and requirements.

Step 4 Delete the row or column which is fully exhausted. Again compute the column and row penalties for the reduce transportation table and then go to step (2). Repeat the procedure until all the rim requirements are satisfied.